THEOREM X.X. *The tile complexity of a self-assembled, just-barely 3D, 2 x k x N rectangle at temperature 1 is O(N^1/k + k) for k>=7 in the abstract tile assembly model.*

The following is a proof by construction. Let *m=*.... *k’=(k-4)/3….* and *m’*, which is *m* rounded up to the nearest multiple of 2. The assembly constitutes a *k’*-digit, base-*m* counter. For the purpose of illustration, we will reference a 2-digit, ternary counter.

We begin by hard-coding the **Seed** gadget [Figure A] with *k’* columns such that each column represents a digit (leading digit first), where a collection of geometric bit-bumps on the columns’ east sides encodes the digits into binary (arbitrarily). These digits are optimally chosen such that the counter stops just before reaching a height of N tiles, at which point, the construction is given a flat “roof” and adds a small number of rows to finish reaching a length of N. A small “lip” may be added on the east side of the **Seed** gadget in case *k-1* is not divisible by 3; this catches the **drop tiles** at the end of the construction. The complexity of the constructions is directly related the **Seed** gadget, which is *O(m’ + k’)*.

After the **Seed** gadget, the **Right\_Wall** gadget [Figure B] attaches to the vacant north-facing glue. The **Right\_Wall** gadget’s purpose is to initiate a row of **Counter** gadgets [Figures C-H], and then to block the **Return** gadget [Figure J] so that a new row can form.

The set of **Counter** gadgets consist of *m* **Increment** gadgets and *m* **Copy** gadgets. Each row of the counter has *k’* **Counter** gadgets and begins by adjoining the west facing glue of the **Right\_Wall** gadget with an **Increment** gadget. Each **Counter** gadget reads over a series of bit-bumps protruding into their row from the preceding **Seed** gadget or counter row. After reading the bit patterns via **guess tiles** [Figure P], the set of possible **Counter** gadgets is narrowed to one, and then a bit pattern is either copied or incremented onto the row above by the **Counter** gadget. The **Increment** gadget that increments *m-1* to *0* is special in that its west-facing glue initiates another **Increment** gadget and its south-facing glue initiates the **End\_Left\_Wall** gadget (whichever glue is not blocked). Other **Increment** gadgets initiate a **Copy** gadget with their west-facing glues and a regular **Left\_Wall** gadget with their south-facing glues. The **Copy** gadgets similarly initiate another **Copy** gadget or a **Left\_Wall** gadget [Figure I].

The purpose of the **Left\_Wall** gadget is to block the construction of additional **Counter** gadgets for the next row, forcing it to produce a **Left\_Wall** or **End\_Left\_Wall** gadget [Figure L]. The preceding counter row will have been blocked by the preceding **Left\_Wall** gadget or the **Seed** gadget. Additionally, the **Left\_Wall** gadget initiates a **Return** gadget with its east-facing glue.

The **Return** gadget ends with a **guess tile** on its east side which produces more **Return** gadgets in that direction, passing over the tiles that were left by the **Counter** gadgets until this row of **Return** gadgets reaches the preceding **Right\_Wall** gadget and is forced to initiate a **Right\_Wall\_Foundation** gadget [Figure K]. The **Right\_Wall\_Foundation** gadget initiates another **Right\_Wall** gadget.

This entire process repeats itself until an **End\_Left\_Wall** gadget is produced, at which point, all **Counter** gadgets will be outputting 0.

The **End\_Left\_Wall** gadget is made tall enough to block all future **shingle tiles** [Figure Q], and then initiates an **End\_Return** gadget [Figure M] from the east-facing glue. The **End\_Return** gadget is similar to the **Return** gadget in that its **guess tile** initiates either another **End\_Return** gadget or the **Roof** gadget. The **End\_Return** gadget is also extend northword in order to cover empty tile spaces that would have been filled by the, now absent, new counter row.

The **Roof** gadget [Figure O] contains a hard-coded tile column that reaches above the protruding tiles from the last counter row and then extend the assembly to a height of N. Each tile in the vertical column that extends past the counter row has a west-facing glue. The west-facing glue accepts a **shingle tile**, which extend the roof westward until blocked by the **End\_Left\_Wall** gadget. The east-facing glue on the **Roof** gadget’s northmost column tile is appended until it reaches (*k-1*, *N-1*). Each tile that extends the roof eastward has a south-facing glue that accepts a **drop tile** [Figure R]. The **drop tiles** will replicate southward until blocked by a **Right\_Wall** gadget or the “lip” on the **Seed** gadget.

An entire 2-digit, ternary example of the counter is illustrated in figure R.

The tile complexity for each gadget is *O(...)*, but since we have *2\*m* **Counter** gadgets, the consolidation of each gadget set brings the complexity of the entire construction to *O(...)*.